14200

Wax

ARPA Network Information Center
Stanford Research Institute
Menlo Park, California 94025
NIC #14200
Packet Radio Temporary Note 23

Peter J. Wong
Stanford Research Institute
February 5, 1973

Channel Utilization of a Slotted Ripple System

Preface*

A stationless network is one of the major alternative architectures now under study. This network form contains only terminals and repeaters, with the latter endowed with sufficient logic and storage to permit various forms of routing in a network of repeaters.

This note is an initial analysis of one aspect of such a network. In particular, the repeaters are assumed to contain only limited directory capabilities, so that a network search must be conducted to find a desired terminal. One such search is expected for every 600 packets generated by a terminal--this envisions a two-minute voice conversation between a pair of terminals, each terminal generating packets at an average rate of 2.5 per second.

The establishment of a conversation begins with the calling terminal launching a "scout" packet to find the called terminal. All succeeding 600 (300 from each terminal) packets can follow directed routes between terminals for the duration of that conversation.

A broadcast scheme is assumed to spread the scout packet. This note analyzes one such scheme, called "slotted ripple." Due to the inefficiency of such a scheme, it is not considered for use in the distribution of normal packets. However, inefficiency for scout packets may prove acceptable. For instance, if a scout packet could be directed through the network as if it were a normal packet, the scout would add a network traffic overhead of 1 to 600. If broadcasting the scout results in a traffic load 10 times that offered, then the scout represents an overhead of 10 to 600--not an overwhelming burden.

Thus, the results presented in this note should be viewed as not only providing insight into a broadcast scheme, but also as a potentially viable scheme, when properly applied.

---

*The preface was written by E. B. Shapiro

ARPA Network Information Center
Stanford Research Institute
Menlo Park, California 94025
NIC #14300

Packet Radio Temporary Note 32

Channel Utilization of a Broadcast

Preface

A statistical network is one of the [illegible]
now under study. This network form
with the latter endowed with satellite
various forms of routing in a network of

This note is an initial analysis of one
In particular, the repeaters are assumed
directory capabilities, so that a network
packets generated by a terminal--this
connection between a pair of terminals
packets at an average rate of 2.5 per

The establishment of a conversation begins
launching a "scout" packet to find the called
600 (300 from each terminal) packets can follow
terminals for the duration of that conversation.

A broadcast scheme is assumed to spread the
note analyzes one such scheme, called [illegible]
inefficiency of such a scheme, it is not
distribution of normal packets. However,
may prove acceptable. For instance, if a
through the network as if it were a normal
add a network traffic overhead of 1 to
results in a traffic load 10 times that
an overhead of 10 to 600--not an overwhelming

Thus, the results presented in this note
only providing insight into a broadcast
viable scheme, when properly applied.

---

* The preface was written by R. H. Shapiro

Channel Utilization of a Slotted Ripple System

I Introduction

In this note we shall evaluate the channel utilization of a slotted ripple system. The approach used herein is a steady-state analysis that is similar to that used by N. Abramson in Ref. 1.*

A slotted ripple broadcast system requires that terminals and repeaters broadcast only in time slots and that repeaters repeat packets from other repeaters only once. The purpose of this single repetition constraint is to prevent copies of a small number of packets saturating the system. It is envisioned that packets will "ripple" outward only once from the source repeater. The single repetition constraint can be implemented by having the repeater store in a short-term memory identification parameters of packets, which it has recently repeated, so that the packets will not be repeated a second time.

A study of the slotted ripple system is important since it is likely that some variation of this system will be a subcomponent in many broadcast system designs. Although a number of simplifying assumptions are made that are not strictly true, the results presented here are useful for three reasons. First, the analysis provides a qualitative understanding of the interaction of the important system parameters. Second, the method of approach used herein is instructive and may be useful in further analysis. Third, approximate "rule-of-thumb" answers to complex problems can be obtained.

---

* Ref. 1. N. Abramson, "THE ALOHA SYSTEM--Another Alternative for Computer Communication," AFIPS Conference Proceedings, Vol. 37, pp. 281-285, November 1970.

Channel Utilization of a Slotted Ripple System

I Introduction

In this note we shall evaluate the channel utilization of a slotted ripple system. The approach used herein is a steady-state analysis that is similar to that used by N. Abramson in Ref. 1.*

A slotted ripple broadcast system requires that terminals and repeaters broadcast only in time slots and that repeaters repeat packets from other repeaters only once. The purpose of this single repetition constraint is to prevent copies of a small number of packets saturating the system. It is envisioned that packets will "ripple" outward only once from the source repeater. The single repetition constraint can be implemented by having the repeater store in a short-term memory identification parameters of packets, which it has recently repeated, so that the packets will not be repeated a second time.

A study of the slotted ripple system is important since it is likely that some variation of this system will be a subcomponent in many broadcast system designs. Although a number of simplifying assumptions are made that are not strictly true, the results presented here are useful for three reasons. First, the analysis provides a qualitative understanding of the interaction of the important system parameters. Second, the method of approach used herein is instructive and may be useful in further analysis. Third, approximate "rule-of-thumb" answers to complex problems can be obtained.

---

* Ref. 1. N. Abramson, "THE ALOHA SYSTEM--Another Alternative for Computer Communication," AFIPS Conference Proceedings, Vol. 37, pp. 281-285, November 1970.

II Preliminary Definitions and Assumptions

In our model we shall keep track of several types of packets. First, we distinguish between packets broadcast by terminals and repeaters. Second, we differentiate between the first-time broadcast of a packet, and the rebroadcast of copies of the original packet. Third, we discriminate between a "regular" packet, which transmits the basic message of the sender, and an "acknowledgement" packet, which tells the sender that its packet has been received.* Finally, for packets broadcast by a repeater, we distinguish between "own" packets generated by terminals in the repeater's area of coverage, and "other" packets generated by terminals in another repeater's area of coverage. When we do not wish to differentiate between packet types we shall simply use the term packet.

The procedures for transmitting a packet are assumed to be as follows. A terminal will rebroadcast a regular or acknowledgement packet (with random delay) as many times as necessary until its parent repeater successfully hears the packet (i.e., an ALOHA type system is used by the terminals to broadcast to the parent repeater). Once the parent repeater hears the packet, the packet is broadcast and repeated by the repeaters using the ripple method. Upon receipt of a regular packet, only the destination terminal sends back an acknowledgement packet (i.e., end-to-end acknowledgement of regular packets). If the source terminal does not receive an acknowledgement packet in response to a regular packet within a specified time, then the source terminal will rebroadcast the regular packet.

Referring to Fig. 1, the following are some of the preliminary definitions and assumptions used in the analysis. (Other definitions and assumptions will be given during the analysis.)

---

*A first-time regular packet corresponds to a "message packet" in Abramson's terminology (see Ref. 1).

II Preliminary Definitions and Assumptions

In our model we shall keep track of several types of packets. First, we distinguish between packets broadcast by terminals and repeaters. Second, we differentiate between the first-time broadcast of a packet, and the rebroadcast of copies of the original packet. Third, we discriminate between a "regular" packet, which transmits the basic message of the sender, and an "acknowledgement" packet, which tells the sender that its packet has been received.* Finally, for packets broadcast by a repeater, we distinguish between "own" packets generated by terminals in the repeater's area of coverage, and "other" packets generated by terminals in another repeater's area of coverage. When we do not wish to differentiate between packet types we shall simply use the term packet.

The procedure for transmitting a packet are assumed to be as follows. A terminal will rebroadcast a regular or acknowledgement packet (with random delay) as many times as necessary until its parent repeater successfully hears the packet (i.e., an ALOHA type system is used by the terminals to broadcast to the parent repeater). Once the parent repeater hears the packet, the packet is broadcast and repeated by the repeaters using the ripple method. Upon receipt of a regular packet, only the destination terminal sends back an acknowledgement packet (i.e., end-to-end acknowledgement of regular packets). If the source terminal does not receive an acknowledgement packet in response to a regular packet within a specified time, then the source terminal will rebroadcast the regular packet.

Referring to Fig. 1, the following are some of the preliminary definitions and assumptions used in the analysis. (Other definitions and assumptions will be given during the analysis.)

---

*A first-time regular packet corresponds to a "message packet" in Abramson's terminology (see Ref. 1).

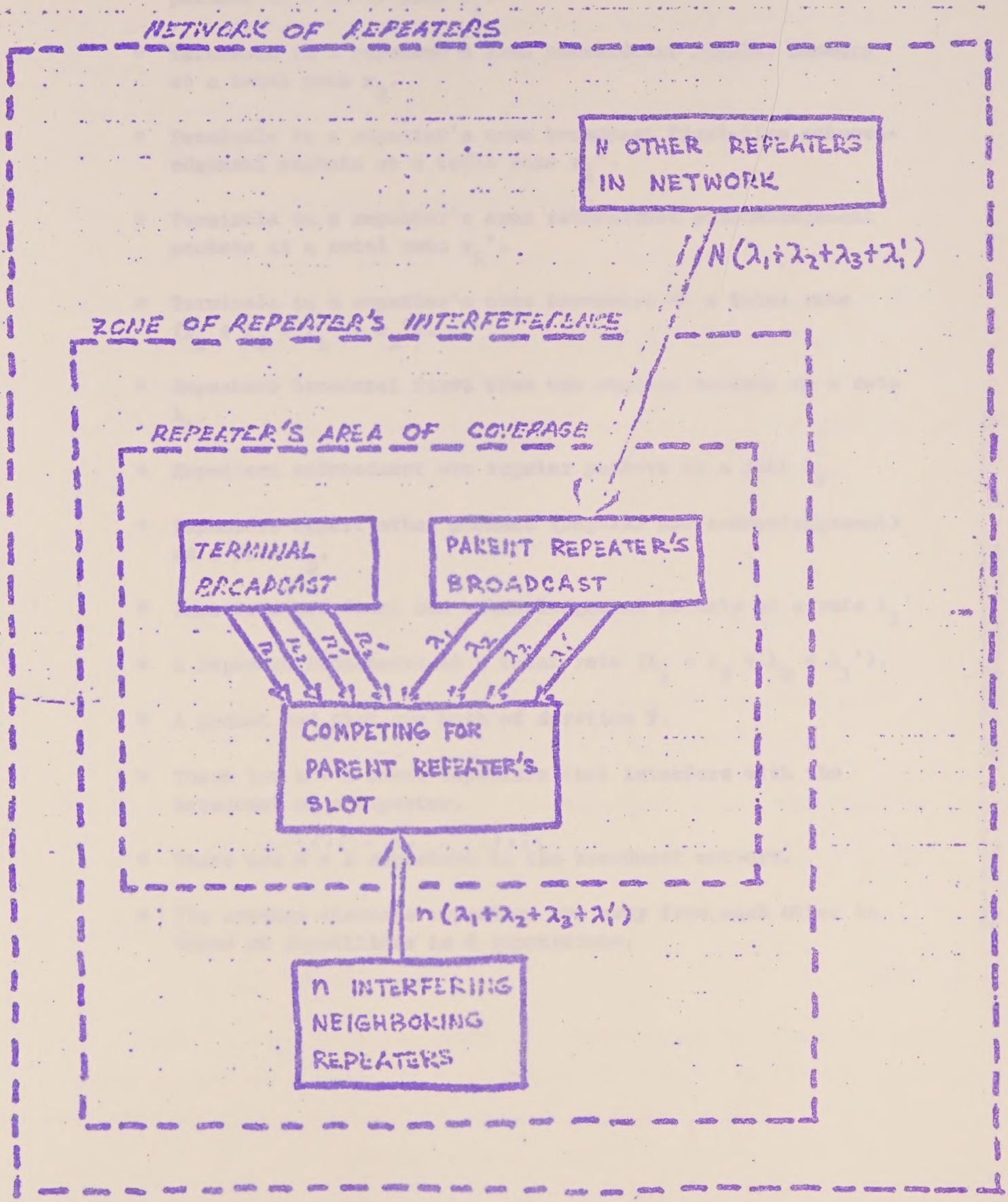
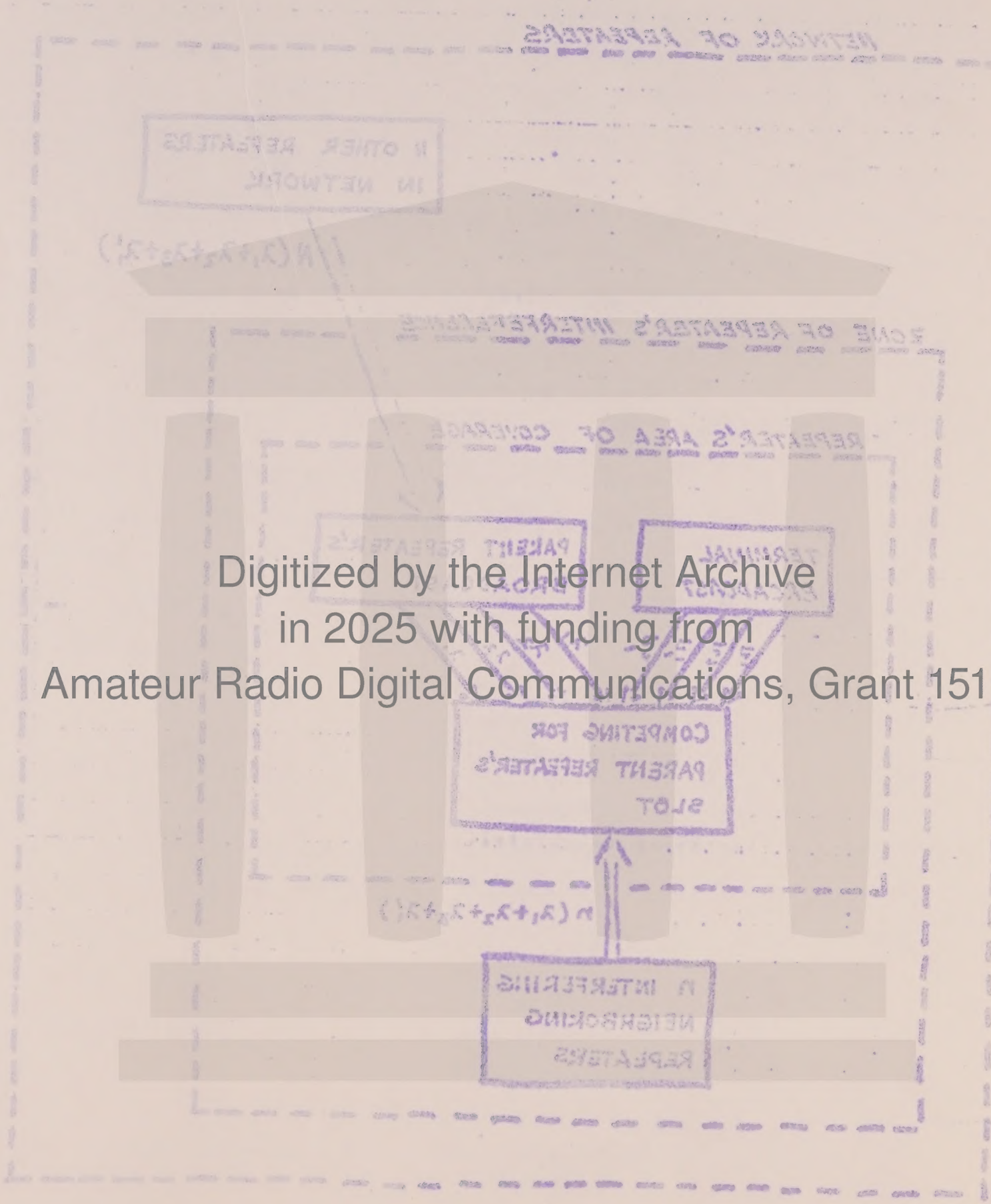


FIG. 1: THE INTERACTION OF THE VARIOUS PACKET RATES



Digitized by the Internet Archive
in 2025 with funding from
Amateur Radio Digital Communications, Grant 151

https://archive.org/details/temporary-note-023

- Terminals in a repeater's area broadcast first-time regular packets at a total rate r_1.
- Terminals in a repeater's area rebroadcast regular packets at a total rate r_2.
- Terminals in a repeater's area broadcast first-time acknowledgement packets at a total rate r_1'.
- Terminals in a repeater's area rebroadcast acknowledgement packets at a total rate r_2'.
- Terminals in a repeater's area broadcast at a total rate $(r_1 + r_2 + r_1' + r_2')$.
- Repeaters broadcast first time own regular packets at a rate λ_1.
- Repeaters rebroadcast own regular packets at a rate λ_2.
- Repeaters repeat other packets (regular and acknowledgement) at a rate λ_3.
- Repeaters broadcast own acknowledgement packets at a rate λ_1'.
- A repeater broadcasts at a total rate $(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_1')$.
- A packet and slot are both of duration T.
- There are n - 1 other repeaters that interfere with the broadcast of a repeater.
- There are N + 1 repeaters in the broadcast network.
- The average distance repeaters are away from each other in terms of repetitions is d repetitions.

- Terminals in a repeater's area broadcast first-time regular packets at a total rate x^I.
- Terminals in a repeater's area broadcast regular packets at a total rate z^3.
- Terminals in a repeater's area broadcast acknowledgement packets at a total rate z^3.
- Terminals in a repeater's area broadcast at a total rate $(z^I + x^3 + z^I + x^3)$.
- Repeaters broadcast first time own regular packets at a rate y^I.
- Repeaters rebroadcast own regular packets at a rate y^3.
- Repeaters repeat other packets (regular and acknowledgement) at a rate y^3.
- Repeaters broadcast own acknowledgement packets at a rate y^I.
- A repeater broadcasts at a total rate $(y^I + y^3 + y^3 + y^I)$.
- A packet and plot are both of duration T.
- There are n - 1 other repeaters that interfere with the broadcast of a repeater.
- There are N + 1 repeaters in the broadcast network.
- The average distance repeaters are away from each other in terms of repetitions in d repetitions.

III Channel Utilization and Channel Traffic

For terminals in a single repeater area, the rate at which they successfully broadcast their basic message packets is r_1. Consequently, from the viewpoint of the terminals in a single repeater area, the channel utilization is

$$\text{CHANNEL UTILIZATION} = r_1 T. \tag{3.1}$$

From the viewpoint of the terminals in single repeater area, the channel traffic is composed of those packets competing for the same slot. Assuming that a terminal broadcasts and interferes over an identical area, and that a repeater broadcasts and interferes over an identical area, we see from Fig. 1 that the following packets are competing for the same slot:

(1) Packets broadcast by the terminals at a rate $(r_1 + r_2 + r_1' + r_2')$

(2) Packets broadcast by the parent repeater at a rate $(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_1')$

(3) Packets broadcast by the (n-1) neighboring interfering repeaters at a rate $(n-1)(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_1')$.

Consequently, the total rate of packets competing for the same slot is

$$R = (r_1 + r_2 + r_1' + r_2') + n(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_1') \tag{3.2}$$

and the channel traffic is

$$\text{CHANNEL TRAFFIC} = RT. \tag{3.3}$$

If we assume exponential packet broadcast times for terminals and repeaters and independence in their broadcast times, then the probability of a packet capturing a slot of duration T is from (3.2)*.

$$P = \Pr(\text{SLOT CAPTURE}) = e^{-RT}. \tag{3.4}$$

*In this note, the notation Pr(E) means the probability that event E occurs, and the notation Pr(E1/E2) means the probability that event E1 occurs, given that event E2 has occurred.

$$R = (x^1 + x^2 + x^1_1 + x^2_1) + n$$

IV Derivation of Expressions

In this section we shall derive expressions relating the channel utilization, $r_1 T$, to the channel traffic, RT. Essentially this will require solving for rates r_2, r_1', r_2', λ_1, λ_2, λ_3, and λ_1' in terms of r_1, R, and other system parameters.

A. Steady-State Considerations

The rate at which terminals in a repeater's area successfully broadcast regular packets and acknowledgement packets is r_1 and r_1' respectively. The rate at which a repeater successfully broadcasts own regular and own acknowledgement packets is λ_1 and λ_1' respectively. However, in steady state, the rate at which a repeater receives packets must equal the rate at which it broadcasts packets; therefore, we have

$$\lambda_1 = r_1, \text{ and} \tag{4.1}$$

$$\lambda_1' = r_1'. \tag{4.2}$$

We assume in steady state that there is an equal probability of all terminals serving as the source and destination of packets (i.e., the traffic distribution is uniform). Consequently, we assume that the terminals in a repeater's area receive regular packets at a total rate r_1; this is the same rate at which the terminals successfully broadcast regular packets. Because a terminal must send an acknowledgement packet for each regular packet it receives, then

$$r_1' = r_1, \tag{4.3}$$

and consequently, using (4.2) we have

$$\lambda_1' = r_1. \tag{4.4}$$

B. Expression for r_2

The rate at which terminals rebroadcast regular packets is r_2, and in steady state this must satisfy the equation

14. Derivation of

[illegible] we shall derive

z_2^T, to the channel input

[illegible] for values z^3, z_1^3, z_2^3

[illegible] given random parameters.

[illegible] Characteristics

[illegible] at which terminals in a

[illegible] regular packets and acknowledg-

[illegible] respectively. Also note at which a

[illegible] regular and own acknowledgement packet

[illegible] however, in steady state, the rate at

[illegible] equal to [illegible] at which it produces

$$y^1 = z_1^3, \text{ and}$$

$$y_1^1 = z_2^3.$$

[illegible] we assume in steady state that there

[illegible] arriving as the source and destin-

[illegible] tic distribution is uniform). Consequ-

[illegible] a repeater's area receive a

[illegible] is also the same rate at which the termi-

[illegible] regular packets. Because a terminal must

for each regular packet it receives, then

$$z_1^3 = z^1,$$

and consequently, using (4.8) we have

$$y^1 = z_1^3.$$

[illegible] Expression for r_2^3

[illegible] rate at which terminals acknowledgements

[illegible] and is simply given that must satisfy the

$$r_2 = (r_1+r_2)\Pr(\text{TERMINAL REBROADCAST REGULAR PACKET}). \quad (4.5)$$

A terminal will rebroadcast a regular packet if either of the following two disjoint events occurs:

EVENT 1: The parent repeater does not receive the regular packet, or

EVENT 2: A) The parent repeater receives the packet, and.

B) no acknowledgement packet is received by the the source terminal.

Because the above events are disjoint, we have

$$\Pr(\text{TERMINAL REBROADCAST REGULAR PACKET})$$

$$= \Pr(\text{EVENT 1}) + \Pr(\text{EVENT 2}). \quad (4.6)$$

The parent repeater will not receive a regular packet if the packet does not capture the slot, consequently,

$$\Pr(\text{EVENT 1}) = 1 - P \quad (4.7)$$

where P is given in (3.4).

EVENT 2 occurs when EVENT 2A and EVENT 2B occurs. Using Baye's Rule, we have

$$\Pr(\text{EVENT 2}) = \Pr(\text{EVENT 2A, EVENT 2B})$$

$$= \Pr(\text{EVENT 2B} \mid \text{EVENT 2A})\Pr(\text{EVENT 2A}). \quad (4.8)$$

The parent repeater will receive a regular packet if the packet captures the slot, consequently,

$$\Pr(\text{EVENT 2A}) = P. \quad (4.9)$$

For simplicity, we assume that regular and acknowledgement packets are repeated along a single chain of repeaters whose distance between the source terminal's parent repeater and the destination terminal's parent repeater is d repetitions. Thus, if the regular packet is received by the parent repeater, then the probability of the source

terminal receiving back an acknowledgement packet is the same as the probability of having $2d + 2$ successful repetitions.* Because the probability of a successful repetition is the same as the probability of capturing a slot, we have

$$\Pr(\text{EVENT 2B} \mid \text{EVENT 2A}) = 1 - P^{2d+2}. \tag{4.10}$$

Using (4.8), (4.9), and (4.10), we have

$$\Pr(\text{EVENT 2}) = (1-P^{2d+2})P. \tag{4.11}$$

Finally, by using (4.6), (4.4), (4.7), and (4.11), we have

$$r_2 = (r_1+r_2)(1-P^{2d+3}). \tag{4.12}$$

Solving (4.12) for r_2 in terms of r_1, we have

$$r_2 = \left(\frac{1 - P^{2d+3}}{P^{2d+3}}\right) r_1 \tag{4.13}$$

C. Expression for r_2'

The rate at which terminals rebroadcast acknowledgement packets is r_2', and in steady state this must satisfy the equation

$$r_2' = (r_1' + r_2')\Pr(\text{TERMINAL REBROADCAST ACKNOWLEDGEMENT PACKET}). \tag{4.14}$$

A terminal will rebroadcast an acknowledgement packet, only if the acknowledgement packet is not received by the parent repeater; this occurs when the acknowledgement packet does not capture the slot. Consequently,

$$r_2' = (r_1' + r_2')(1-P) \tag{4.15}$$

where P is given by (3.4). By substituting (4.3) into (4.15) we can solve for r_2' in terms of r_1, i.e.,

$$r_2' = \left(\frac{1-P}{P}\right) r_1. \tag{4.16}$$

*The two extra repetitions account for the broadcast of a repeater to a terminal at the destination and source ends of the chain.

D. Expression for λ_2

The rate at which repeaters rebroadcast own regular packets is λ_2, and in steady state this must satisfy the equation

$$\lambda_2 = (\lambda_1 + \lambda_2)\Pr(\text{REPEATER REBROADCAST OWN REGULAR PACKET}). \quad (4.17)$$

A repeater will rebroadcast its own regular packet, if a source terminal in its area does not receive an acknowledgement packet corresponding to the own regular packet broadcast by the repeater. Again we assume that regular and acknowledgement packets are repeated along a single chain of repeaters whose distance between the source terminal's parent repeater and the destination terminal's parent repeater is d repetitions. Consequently, the probability that a repeater will have to rebroadcast one of its own regular packets is the same as the probability of not having $2d + 2$ successful repetitions, i.e.,

$$\Pr(\text{REPEATER REBROADCAST OWN REGULAR PACKET}) = 1 - P^{2d+2}, \quad (4.18)$$

where P is given in (3.4). By substituting (4.18) into (4.17) we have

$$\lambda_2 = (\lambda_1 + \lambda_2)(1 - P^{2d+2}). \quad (4.19)$$

By substituting (4.1) in (4.19), we can solve for λ_2 in terms of r_1, i.e.,

$$\lambda_2 = \left(\frac{1-P^{2d+2}}{P^{2d+2}}\right) r_1. \quad (4.20)$$

It is to be remarked that $\lambda_2 < r_2$, since the terminal's rebroadcast rate r_2 contains a component not contained in the parent repeater's rebroadcast rate λ_2. More specifically, the rate r_2 contains the terminal's rebroadcast of regular packets necessary to insure that the parent repeater hears the regular packet; this essentially corresponds to EVENT 1 as defined in Section IVB. By comparing (4.10) and (4.18) we see that

$$\Pr(\text{REPEATER REBROADCAST OWN REGULAR PACKET})$$

$$= \Pr(\text{EVENT 2B} \mid \text{EVENT 2A}), \tag{4.21}$$

where EVENT 2A and EVENT 2B are defined in Section IV B.

E. Expression for λ_3

The rate at which repeaters repeat other packets (regular and acknowledgement) is λ_3. Now, a repeater will repeat another repeater's packet only once; this is the ripple assumption. Because of this assumption, we assume that a repeater at most repeats only once the own packets (regular and acknowledgement) of other repeaters. However, repeaters are broadcasting own packets at a rate $(\lambda_1 + \lambda_2 + \lambda_1')$, and consequently, the N other repeaters are broadcasting own packets at a total rate $N(\lambda_1 + \lambda_2 + \lambda_1')$. If a repeater receives these $N(\lambda_1 + \lambda_2 + \lambda_1')$ packets from other repeaters, it must repeat these packets. Therefore in steady-state we have

$$\lambda_3 = N(\lambda_1 + \lambda_2 + \lambda_1')\Pr(\text{REPEATER RECEIVES OTHER REPEATER'S OWN PACKETS}). \tag{4.22}$$

We assume that the other N repeaters are at a distance of d repetitions away, and that packets are transmitted along a single chain of repeaters of length d repetitions. Then the probability of a repeater receiving another repeater's own packet, is the probability of having d successful repetitions, i.e.,

$$\Pr(\text{REPEATER RECEIVES OTHER REPEATER'S OWN PACKET}) = P^d, \tag{4.23}$$

where P is given by (3.4). Substituting (4.23) into (4.22), we have

$$\lambda_3 = N(\lambda_1 + \lambda_2 + \lambda_1')P^d. \tag{4.24}$$

Using (4.1), (4.4), and (4.20), we can solve for λ_3 in terms of r_1, i.e.,

$$\lambda_3 = N\left(\frac{1 + p^{2d+2}}{p^{d+2}}\right) r_1. \tag{4.25}$$

where p is given by (3.4). Substituting

$$= p_q^{\cdot}$$

Using (4.1), (4.4), and (4.20), we can solve

$$y^3 = N(y^1 + y^3 + y^1)p_q^{\cdot}$$

$$y^3 = \left(\frac{\quad}{\quad} \right)$$

F. Calculating Channel Utilization

Previously we have obtained r_2, r_1', r_2', λ_1, λ_2, λ_3, and λ_1' in terms of r_1 and other system parameters. We would like to obtain an expression for the channel utilization r_1T as a function of the channel traffic RT. To do this we substitute (4.1), (4.3), (4.4), (4.13), (4.16), (4.20), and (4.25) into (3.2); we multiply both sides of (3.2) by T and solve for r_1T as a function of RT. Doing this, we have

$$r_1T = \left[\frac{P^{2d+2}}{1+P^{2d+2} + nP(1+P^{2d+2}) + nNP^{d+1}(1+P^{2d+2})} \right] RT, \qquad (4.26)$$

where P is given by (3.4).

Channel Utilization

$$\left| \frac{\qquad\qquad}{\qquad\qquad} \right| = T_{\cdot}$$

is given by (3.4).

V Analysis of Results

In Figures 2A and 2B we have plotted the channel traffic, RT, versus the channel utilization, $r_1 T$, for a 37 repeater network in a hexagonal pattern and a 2 repeater network respectively.* (For the 37 repeater network N = 36, n = 6, and d = 4; for the 2 repeater network N = 1, n = 2, and d = 1.) In both cases, we see the unstable saturation behavior exhibited by Abramson in Ref. 1. We note that in both cases the maximum channel utilization is very small. The reason for the sad behavior of this broadcast system is that most of the channel traffic is composed of copies of first-time regular packets (i.e., copies of message packets) due to terminals rebroadcasting packets (e.g., $r_2 T$ and $\lambda_2 T$) and repeaters repeating packets (e.g., $\lambda_3 T$).

*In Figs. 2A and 2B we have also plotted $(N+1)RT$ vs. $(N+1)r_1 T$, since, in some sense, these quantities represent respectively the channel traffic and the channel utilization of the network when considered from a network viewpoint. It is also to be noted that for the 2 repeater case, the approximation used in this analysis of a single-chain of repeaters between source and destination terminal of length d repetitions is exact.

V Analysis of

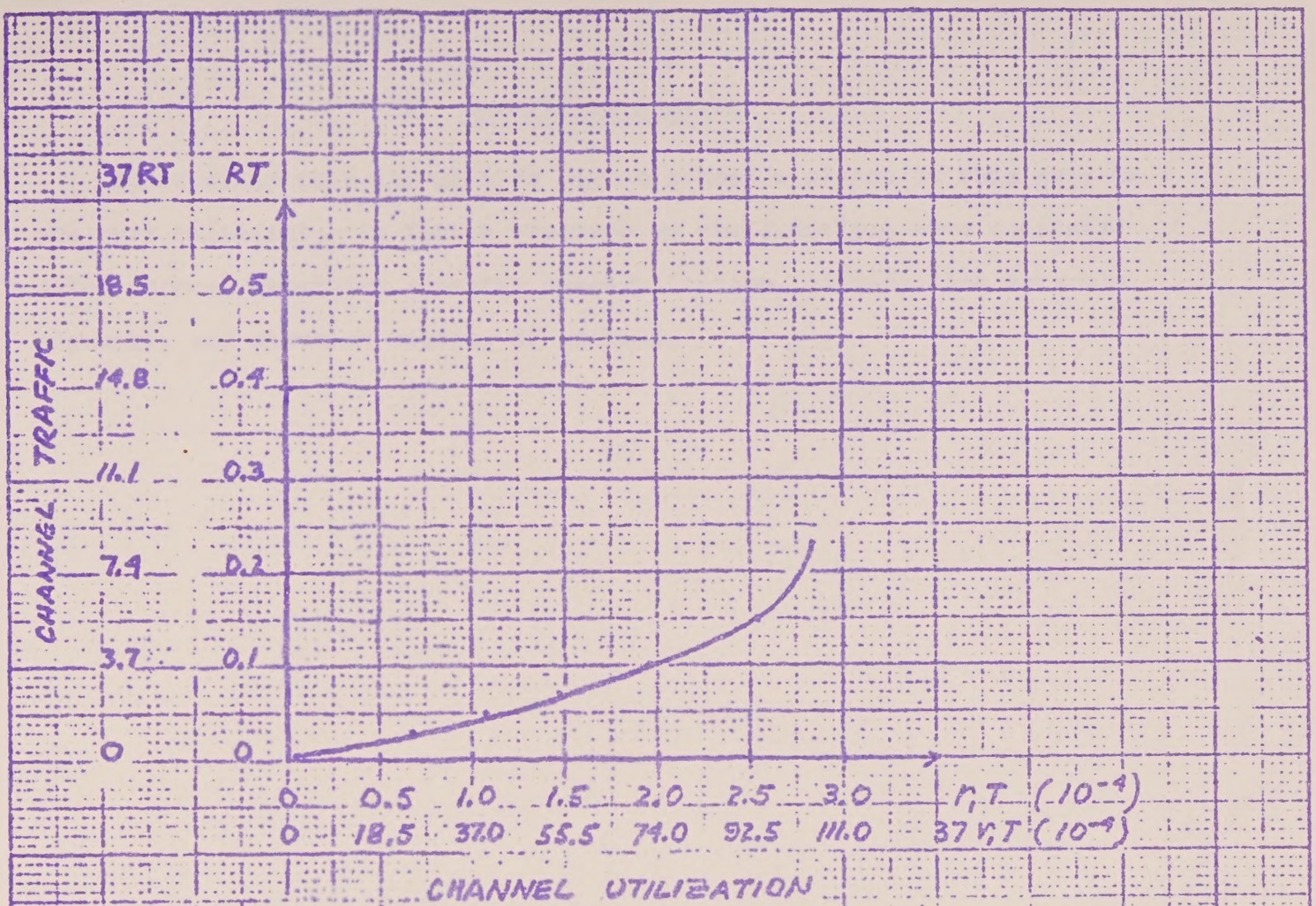


FIG. 2A: CHANNEL TRAFFIC (RT) VS. CHANNEL UTILIZATION (r,T) -- 37 REPEATERS

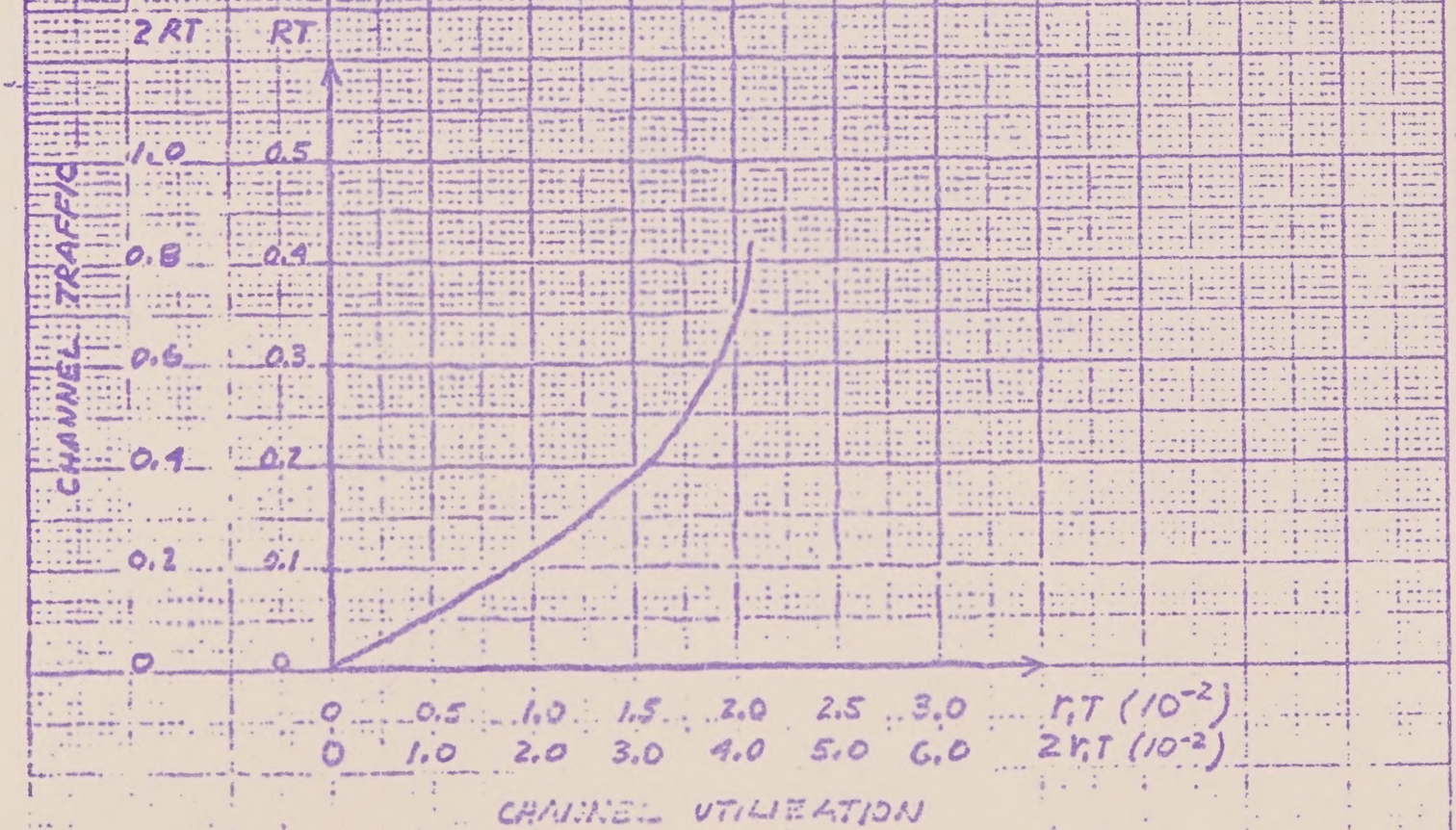
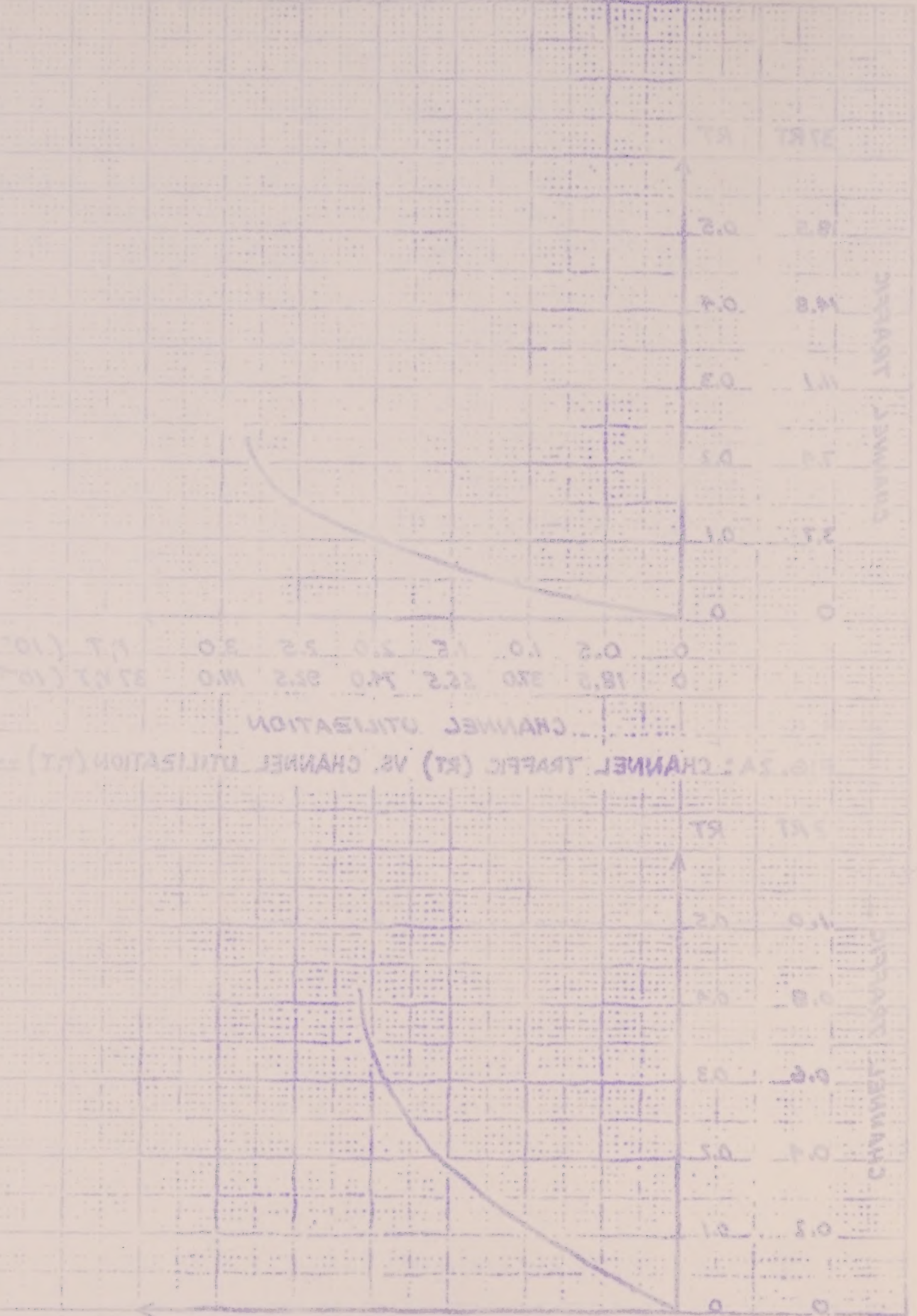


FIG. 2B: CHANNEL TRAFFIC (RT) VS. CHANNEL UTILIZATION (r,T) -- 2 REPEATERS


RT
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1.0
1.5
2.0
2.5
3.0
CHANNEL UTILISATION

VI Concluding Remarks

It is clear that the slotted ripple scheme analyzed in this note is not very efficient. In future research we shall analyze more sophisticated schemes involving repeater synchronization, link-by-link acknowledgement, and directed routing of packets.

VI Concluding Remarks

It is clear that the slotted ripple schemes analyzed in this note is not very efficient. In future research we shall analyze more sophisticated schemes involving repeater synchronization, link-by-link acknowledgement, and directed routing of packets.